Finite element analysis of the relationships between dynamic, physical and mechanical egg properties

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Abbreviated title: FEA of the Egg

Summary

This paper deals with the numerical investigation of the structural-acoustic behaviour of a chicken egg. A three-dimensional finite element model was developed to simulate both the dynamic behaviour of the eggshell and the fluid loading of the inside fluid. The aim of the paper is to analyse the effects of different variations of certain geometrical and material parameters of the model on the structural-acoustic frequency response functions. It has been found that geometrical modifications (eggshell thickness, size of the egg) have a considerable influence on the fluid-structure coupled natural frequencies. In general, variations of material characteristics do not have much influence on the dynamic behaviour. However, the Young's modulus of the eggshell strongly affects the natural frequencies of the coupled system. The obtained results are used to interpret experimentally observed relationships.

Keywords: finite element analysis, structural-acoustic interaction, egg vibration, fluid filled egg

Introduction

Nowadays, the consumer's primary concern is the quality and safety of the food. In the case of consumption eggs, a whole set of methods were developed in the past to measure eggshell properties which are related to eggshell strength. The eggshell strength is regulated by a certain number of variables like the genetic origin, the age of the laying hen, environmental factors as feed composition, diseases, climate conditions and management of the farmer (Solomon, 1991). As such, accurate, automated and non-destructive measuring methods for assessing eggshell strength characteristics are an important area of research.

Classically, the eggshell strength is evaluated by means of a non-destructive, quasi-static compression test (Voisey and Hunt, 1974). The egg is placed horizontally between two parallel steel plates and a compression load is exerted on the object. Force and displacement are recorded throughout the test and used to calculate the static stiffness (k_{stat}). The slope of the force-displacement curve provides a measure of the eggshell stiffness. However, this method is time-consuming and requires expensive test equipment. The method can not be included in an industrial tool for real-time assessment of shell strength. As an alternative to traditional techniques, Coucke et al. (1994) introduced a dynamic test method for eggshell stiffness assessment. Based on an experimental modal analysis of a chicken egg, the dynamic behaviour was characterized. Several interesting spherical modes are detected in the frequency range 3-8 kHz. The mode shapes in this frequency range all show maximum deformation at the equator of the egg, while the sharp and blunt poles are immobile. The mode shape of the first, flexural spherical mode (S_{20}) shows an oblate-prolate deformation at the equator. The damped natural frequency of this mode and the total egg mass are used to calculate the dynamic eggshell stiffness (k_{dyn}). Modelling the egg as a mass-spring system, the dynamic stiffness k_{dyn} is given as:

$$k_{dyn} = cte \cdot m \cdot RF^2$$

with m the mass of the egg, cte a constant (set to 1) and RF the resonant frequency of the first spherical mode (S_{20}). Thanks to this work, the dynamic behaviour of a chicken egg is assessed at present by using the Acoustic Egg Tester (AET), a lab-scale device for the measurement of the dynamic stiffness of the eggshell based on the identification of the mode shapes, the resonant frequencies and corresponding damping ratios of these modes by the interpretation of the vibration response of an egg excited at its equator with a non-destructive impact. This technique can also be used to detect cracks in the eggshell (De Ketelaere et al., 2000).

Furthermore, the relationships between some of the physical and mechanical egg and eggshell quality parameters and the dynamic, mechanical properties of a chicken egg have been evaluated in many studies. However, little information is available about the contribution of the basic material and geometrical properties to the result of the dynamic tests. No full explanation could be found for the different effect of size and eggshell thickness on the mechanical behaviour. Coucke (1998) utilized simple structural models to simulate the dynamic mechanical behaviour of the egg using the finite element method. The analysis was incomplete as the egg content, i.e. the interior fluid, was not incorporated in the model. Since the content effect was neglected, the analysis results showed several deficiencies when comparing numerical and experimental data. In spite of some important observations achieved using the above-mentioned structural models, there is still a wide gap between such models including only the eggshell and highly detailed structural-acoustic models incorporating both eggshell and fluid content. In the light of the above, the main aim of this paper is to set up a realistic model for an egg structural-acoustic analysis, useful to assess, visualise and compare structural-acoustic behaviour of the egg for different material and geometrical properties. The objective of this parametric study is to investigate the influence of gradual changes in different parameters of the egg model on structural-acoustic frequency response functions. The developed approach and the obtained results would be of interest to specialists working on acoustic based egg grading machines. It is also expected that this approach allows an improved interpretation of the experimentally observed correlations between egg and eggshell parameters.

Problem definition

The avian egg is a biological structure of high complexity. It contains an air chamber and a viscous liquid surrounded by two membranes and an external covering called the eggshell. The base numerical model used in the simulation studies represents a simplified replica of a chicken egg, a fluid filled shell, yielding a coupled structural-acoustic problem. The eggshell is modelled as a single layer shell structure of uniform thickness. The acoustic content includes the air chamber and water, the major components of albumen (~ 90 %) and yolk (~ 50 %). The shell membranes are not incorporated in the model.

Vibrating structures induce acoustic pressure waves in a connected fluid and, vice versa, acoustic pressure waves act as external loads yielding structural vibrations. This fully coupled structural-acoustic problem description is a thoroughly investigated field of research (Fahy, 1985). The numerical approach used in this paper for the representation of the coupling effects between fluid and structure is based on a Finite Element (FE) representation of the structure as well as the fluids (Zienkiewicz et al., 2005). The main advantage of such a method is that is easily possible to represent in one model cavities with different types of fluid, e.g. water and air (Stavrinidis et al., 2001).

Description of the base model

The base model represents a simplified replica of a chicken egg. The overall dimensions of the model are 4.6, 5.8 and 4.6 cm, respectively in X (longitudinal), Y (vertical) and Z (lateral) direction. The eggshell thickness is assumed to be uniform over the shell surface. A default value of 0.38 mm is applied. The material parameters of the eggshell are as follows: Young's modulus $E = 3 \cdot 10^{10}$ N/m^2, Poisson's ratio $\nu = 0.307$ and the mass density $\rho = 2400$ kg/m^3 (Bain, 1992). The egg content is represented by an air chamber and a water domain. The height of the air chamber for the default configuration is 4 mm. The acoustic parameters of the air are: speed of sound 343 m/s and the mass density 1.25 kg/m^3. The default values for the acoustic parameters of water are: speed of sound 1500 m/s and the mass density 997 kg/m^3.

The finite element meshes (Figure 1) for both structural and acoustic domain are generated using MSC.Patran. All uncoupled structural results are obtained with the MSC.Nastran software, while the acoustic and coupled vibro-acoustic results are obtained with the LMS.Sysnoise software. Using mesh morphing techniques, the base meshes are modified for the various analysis (see below). As such all models comprise exactly the same number of nodes and elements. For the structural part of the analysis, bilinear four-noded quadrilateral and three-noded triangular shell elements (6 degrees of freedom per node) are used, whereas for the acoustic part linear eight-noded hexahedral and six-noded wedge elements (1 degree of freedom per node) are employed. The applied element size is consistent with the rule of thumb that states that at least six linear elements should be used per wavelength to assure sufficient prediction accuracy. The maximum frequency of interest is selected as 5000Hz yielding an element size of 0.0025m, which results in 4920 acoustic and 1050 structural elements.

The structural-acoustic model involved in the simulations is a free boundary condition model excited by a unit normal point force exerted at the egg equator (Figure 1).

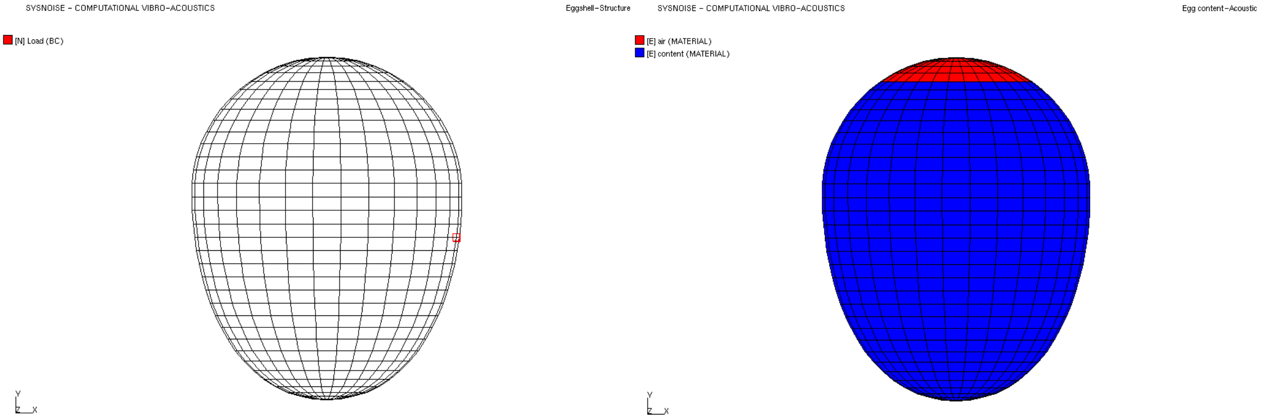

Figure 1. Finite element mesh of the eggshell (left) and of its acoustic content (right)

Results and discussion

The first 50 structural modes are calculated. The eigenfrequencies of these uncoupled structural modes are in general higher than the experimentally observed values. This is expected since only the eggshell is modelled and the egg content, which represents 85-90 % of the total egg mass, is not yet incorporated in the structural model. However, for a fluid filled egg, the eigenfrequencies of the coupled modes are close to the experimental results. The mode shapes and the sequence of appearance of the calculated modes are very similar to the experimentally observed modes. Figure 2 represents a top view (left) and a front view (right) of the mode shape of the first, flexural spherical mode.

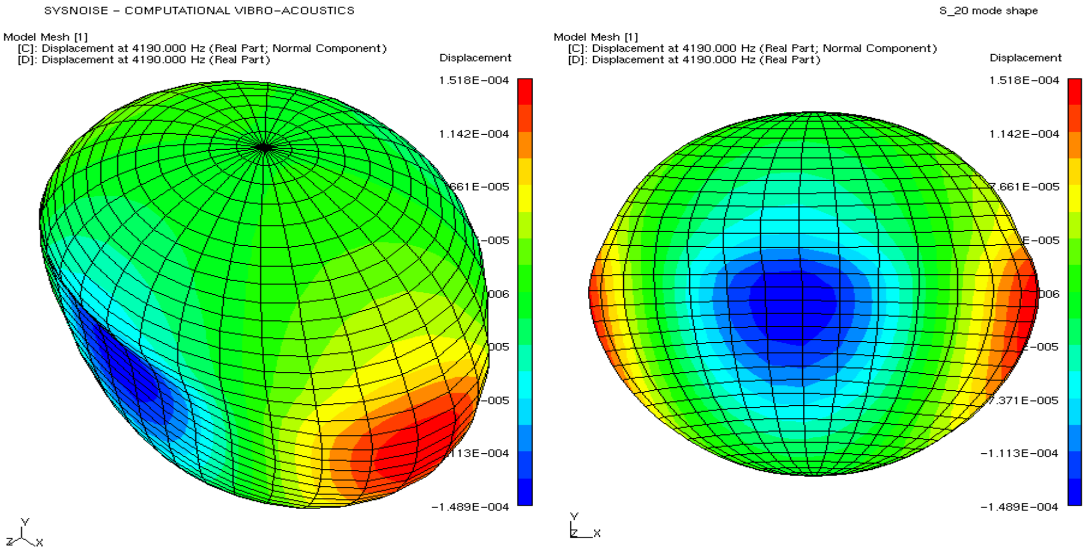

Figure 2. Top and front view of the mode shape of S_{20} mode

The first flexural mode has an eigenfrequency of 4190 Hz and it is called the oblate-prolate mode. The sharp and blunt sides of the egg do not vibrate. All deformation is concentrated towards the equator zone of the egg. An elliptic shape can be recognized at the equator ring. The amount of deformation decreases gradually towards the poles (blunt and sharp ends) of the egg. In the subsequent analysis each single geometrical and material property is varied within a reasonable range. The effect of these changes on the eigenfrequency of the first flexural mode (S_{20}) is evaluated.

Effects of geometrical variations

Variations of the air chamber height: The eigenfrequency of the S20 mode is almost not influenced by the air chamber height. The results can be explained by the opposite effect of the egg mass on its natural frequency. An increase in air chamber height reduces the total egg mass and therefore the natural frequency of the structure will slightly increase.

Variations in the model size and in the eggshell thickness: In order to study the effect of geometrical parameters on the structural-acoustic frequency response, several pairs of egg size and eggshell thickness have been considered. The size of the base model was varied between 10% less and 20% more, whereas the associated eggshell thickness ranged between 0.25 mm and 0.5 mm. The eigenfrequency of the S20 mode increases with the eggshell thickness and is inversely proportional to the size of the egg. Generally, a thicker shell has higher frequencies of vibration. This is because an increase in shell thickness strengthens the shell stiffness and this increases the natural. In the same time, a reduction in size of the egg reduces its mass and, consequently, the resultant resonant frequency will increase.

Effects of material variations

Variations in the Young modulus of the shell material: The eigenfrequency of the first flexural mode (S20) was calculated for six different variations of the Young modulus of the shell. One can notice a major influence of the Young modulus of the shell material on the frequency response of the model. Typically, the Young modulus (E) is a measure of the stiffness of a material. A reduction in E modulus decreases the eggshell stiffness and this reduces the natural frequency of the global system.

Variations in the Poisson ratio of the eggshell: Commonly, Poisson's ratio refers to a characteristic dimensionless number which accurately predicts the amount of deformation experienced in non-parallel directions to an applied load. For most materials, this value ranges between 0.0 and 0.5. Note that Poisson's ratio should not be confused with stiffness or hardness. Materials with similar Poisson's ratio may have completely different Young's modulus. It should be noticed that the increase in Poisson's ratio leads to a minor diminution in resonant frequency. Thus, a variation of Poisson's ratio in a reasonable interval from 0.25 to 0.4 has almost no effect on frequency. A similar effect was observed by Coucke et al. (1998) in the finite element analysis of structural eggshell models.

Variations of the eggshell density: If the eggshell density is increased, the eigenfrequency of the first mode will lightly decrease. For example an increase from 1800 kg/m3 to 2800 kg/m3 will decrease the first flexural frequency from 4240 Hz to 4155 Hz. Furthermore, one can notice that the eigenfrequency of the S20 mode varies inversely to the shell density.

Variations in the albumen density and in eggshell thickness: So far, the individual effect of geometrical or material parameters on the eigenfrequency has been studied. However, in this section we focus on the frequency response of the egg model as a function of combined geometry and material property modifications. Therefore, the density of albumen has been varied between 600 kg/m3 and 1400 kg/m3, whereas the eggshell thickness interval ranged from 0.25 mm to 0.5 mm. It was observed that the eigenfrequency of the S20 mode decreases with increasing internal fluid density. Thus, for any given thickness value, the more dense the albumen, the lower the resonant frequency. On the other hand, an increase in eggshell thickness increases the structural stiffness and this will raise the resonant frequency.

Conclusions

In the present paper, a comprehensive parametric analysis of the structural-acoustic frequency response of a realistic model of chicken egg has been carried out using the finite element method. Some important geometrical parameters of the model, such as height of the air chamber, size of the egg and eggshell thickness, have been varied in order to determine their influence on eigenfrequency of the first flexural mode. Furthermore, different material parameters have been used to model the egg, and their effect on frequency response has been analysed. It has been found that geometrical modifications considered in this study have a major influence on the structural-acoustic response of the egg. In contrast to the above mentioned variations, not all the material parameters have much influence on the dynamic behaviour of the model. Mostly the Young's modulus of the eggshell as well as the albumen density strongly affects the eigenfrequency of the global system. The present work demonstrates the computability of the suggested approach. Further investigations will be carried out to better understand the previous results and experimental measurements will be conducted in order to validate the current numerical analysis.

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